QUESTION 1

If any of the following limits do not exist, assign a value of -37 to that letter.

$$\begin{aligned}
A &= \lim_{x\to\infty} \frac{3x^2+2x+1}{4x^2+6x+7} \\
B &= \lim_{x\to 3} \frac{x^2-x-6}{x^2-5x+6} \\
C &= \lim_{x\to 4} \frac{x-4}{\sqrt{x}-2} \\
D &= \lim_{x\to 1000} 7
\end{aligned}$$

Find $4A+B+C+3D$.

QUESTION 2

Let

$$\begin{aligned}
A &= \text{the sum of the coefficients of } \frac{dy}{dx}(\sin(4e^{i\cos(\frac{15\pi}{4})})) \\
B &= \frac{dy}{dx}(\sin(x)\cos(x)) \text{ at } x = \frac{\pi}{6} \\
C &= \frac{dy}{dx}(y = x^{\cos(x)}) \text{ at } x = \pi \\
D &= \frac{d^2y}{dx^2}(\sin(x)\cos(x)) \text{ at } x = \frac{\pi}{2}
\end{aligned}$$

Find $A + B + C\pi^2 + D$.

QUESTION 3

Given the function $f(x) = e^x$, evaluate the following:

$$
\begin{aligned}
A &= \text{the area bounded by } f(x) \text{ and the x-axis from 0 to 2} \\
B &= \text{the slope of the tangent line to } f(x) \text{ at x} = 1 \\
C &= \text{the volume of the solid formed by rotating } f(x) \text{ around the y-axis bounded by } x = 0 \text{ and } x = 2 \\
D &= \text{the volume of the solid formed by rotating } f(x) \text{ around the x-axis bounded by } x = 0 \text{ and } x = 2
\end{aligned}
$$

Find $A + B + \dfrac{C}{2\pi} + \dfrac{2D}{\pi}$.

QUESTION 4

Given

x	-3	0	1	6	7	8	10	12	15
$f(x)$	2	13	-12	5	2	0	-12	2	3
$g(x)$	3	0	5	-6	3	1	5	6	9

$f(x)$ is a ninth degree polynomial with leading coefficient 2. The Left-Hand Riemann sum of $f(x)$ from 0 to 15, using five equal intervals, is 15.
$g(x)$ is a tenth degree polynomial with leading coefficient 3 with one distinct root in common with $f(x)$. The Left-Hand Riemann sum of $g(x)$ from 0 to 15, using five equal intervals, is also 15.

$$
\begin{aligned}
A &= \text{the right-hand Riemann sum of } f(x) \text{ from -3 to 9, using 4 equal subintervals} \\
B &= \text{the midpoint Riemann sum of } f(x) \text{ from 0 to 15, using 5 equal subintervals} \\
C &= \text{the left-hand Riemann sum of } g(x) \text{ from -3 to 12, using 5 equal subintervals} \\
D &= \text{the midpoint Riemann sum of } g(x) \text{ from 0 to 15, using 5 equal subintervals}
\end{aligned}
$$

Find $A + B + C + D$.

QUESTION 5

Let

$$
\begin{aligned}
A &= \int_5^2 9x^2 + 4x + 5\,dx \\
B &= \int_e^{e^2} \ln(x)\,dx \\
C &= \frac{d}{dx}\int_5^{2x} e^{\frac{t}{2}} \sin(\frac{t}{2})\,dt \text{ at } \frac{\pi}{2} \\
D &= \int_0^{\frac{\pi}{2}} e^x \sin(x)\,dx
\end{aligned}
$$

Find $A + B + C - 4D$.

QUESTION 6

Start with $x = 0$. For each of the following statements, if it is true add 5 to x. If it is false, subtract 10 from x.

1. Continuity implies differentiability
2. The derivative of the product of two functions is equivalent to the product of their derivatives
3. If $f(x)$ is continuous on some interval, then $f(x)$ attains some maximum and minimum value on that interval
4. The $\lim_{x\to 0} \dfrac{\sin(x)}{x} = \infty$
5. If f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), and $f(a) = f(b)$, then there is a number c in (a, b) such that $f'(c) = 0$
6. If a function is a one-to-one continuous function defined on an interval, then its inverse function is also continuous

QUESTION 7

Let $h(x)$ be equal to the inverse of $f(x)$. Let

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	2	8	6	42
3	4	1	$\frac{1}{6}$	5
5	6	$\frac{1}{4}$	-12	$\frac{1}{3}$

$$\begin{aligned} A &= \frac{d}{dx}\left[f(g(x))\right]\Big|_{x=3} \\ B &= \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]\Big|_{x=5} \\ C &= \frac{d}{dx}g(x) + f'(x) - g'(x)\Big|_{x=3} \\ D &= \frac{d}{dx}h(x)\Big|_{x=2} \end{aligned}$$

Find $A + B + \dfrac{C}{D}$.

QUESTION 8

If the integral diverges, assign a value of 7 to that letter:

$$
\begin{aligned}
A &= \int_1^\infty \frac{dx}{x^2} \\
B &= \int_{-\infty}^\infty \frac{dx}{x^2+1} \\
C &= \int_\infty^0 (1-x)e^{-x}\,dx \\
D &= \int_0^\pi \sec^2(x)\,dx
\end{aligned}
$$

Find $A+B+C+D$

QUESTION 9

If the answer to a letter part is 0, let it be equal to 5, and if the answer to a letter part is infinity, let it be equal to 7.

- The Sierpinski carpet is a two dimensional fractal where the first iteration is a unit square. The second iteration subdivides the square into 9 equal squares and removes the centermost one. Each subsequent case divides all squares into 9 equal squares and removes the center.
- The Menger Sponge is a three dimensional fractal. The first case is a unit cube. Each subsequent case removes parts of the cube such that each face of the n^{th} Menger Sponge is the n^{th} Sierpinski carpet.
- Gabriel's horn is the solid formed by rotating the function $f(x) = \dfrac{1}{x}$ about the x - axis, with the domain $x \leq 1$.

$$\begin{aligned}
A &= \text{the area of the } n^{\text{th}} \text{ iteration of the Sierpinski Carpet, where } n \text{ tends toward } \infty \\
B &= \text{the area of the } 3^{\text{rd}} \text{ iteration of the Sierpinski Carpet} \\
C &= \text{the surface area of the } n^{\text{th}} \text{ Menger Sponge, where } n \text{ tends toward } \infty \\
D &= \text{the volume of the } n^{\text{th}} \text{ Menger Sponge, where } n \text{ tends towards } \infty \\
E &= \text{the surface area of Gabriel's horn} \\
F &= \text{the volume of Gabriel's horn}
\end{aligned}$$

Find $A + 81B + C + D + E + F$.

QUESTION 10

Let

$$
\begin{aligned}
A &= \text{the average value of the function } f(x) = 3x^2 + 4x + 7 \text{ on the interval } [1,4] \\
B &= \text{the value } c \text{ that satisfies the Mean Value Theorem for Derivatives on the interval } [\tfrac{1}{2}, 2] \text{ for the function } f(x) = \frac{x+1}{x} \\
C &= \text{the value } c \text{ that satisfies Rolle's Theorem on the interval } [0,3] \text{ for the function } f(x) = x^2 - 3x \\
D &= \text{the average rate of change of the function } f(x) = x^2 + 3x \text{ over the interval } [1,5]
\end{aligned}
$$

Find $A + B + 2C + D$.

QUESTION 11

According to the quantum mechanical model of the atom, the position of electrons is not governed by strict orbital paths, but rather by a probability distribution function that states the probability of an atom of any given position. For the hydrogen atom in the $1s$ orbital, the probability distribution function can be described purely in terms of r, and is $P(r) = \left(\frac{4r^2}{a_0^3}\right) e^{-2r/a_0}$, where r is the distance of the electron from the nucleus and a_0 is a constant known as the Bohr radius (the position of the electron in the ground state of hydrogen according to the Bohr model of the atom). Then let

A = the distance from the nucleus that the electron is most likely to be in

B = the probability that the electron is outside of the Bohr radius

C = the average distance of an electron from the nucleus

D = the uncertainty of the distance of the electron; this is found by evaluating $\Delta r = \sqrt{\langle r^2 \rangle - \langle r \rangle^2}$, where $\langle r^2 \rangle$ is the average square of the distance of the electron, and $\langle r \rangle$ is the average distance of the electron

Find $A + Be^2 + C + D$.

QUESTION 12

There are a number of means out there as a means (hah) of interpolating a set of nonnegative real numbers, such as the arithmetic mean and the harmonic mean. A general form of means that describes both the arithmetic mean and the harmonic mean (among others) is the power mean $M_p(x_1, x_2, \ldots, x_n)$, where

$$M_p(x_1, x_2, \ldots, x_n) = \left(\frac{1}{n}\sum_{i=1}^{n} x_i^p\right)^p.$$

There is also another generalized type of mean, the Lehmer mean $L_p(x_1, x_2, \ldots, x_n)$, where

$$L_p(x_1, x_2, \ldots, x_n) = \frac{\sum\limits_{i=1}^{n} x_i^p}{\sum\limits_{i=1}^{n} x_i^{p-1}}.$$

Then if we let $\{x_i\}_{i=1}^{2015}$ be defined such that $x_i = 2^i$, let

$$\begin{aligned}
A &= \lim_{p\to\infty} M_p(x_1, x_2, \ldots, x_{2015}) \\
B &= \lim_{p\to-\infty} M_p(x_1, x_2, \ldots, x_{2015}) \\
C &= \lim_{p\to 0} M_p(x_1, x_2, \ldots, x_{2015}) \text{ (hint: L'Hôpital's)} \\
D &= \lim_{p\to\infty} L_p(x_1, x_2, \ldots, x_{2015})
\end{aligned}$$

Find $\log_2\left(\dfrac{CD}{AB}\right)$.

QUESTION 13

The following two statements can be proved using the same theorem. Name that theorem.

1. At any moment in time, there is always at least one location on the earth's surface where the temperature is exactly the same as at the location diametrically opposite on the other side of the globe.
2. If you rotate a table that is placed on an uneven floor about its center, you will always find an orientation where the table is perfectly stable.

QUESTION 14

Let

$$\begin{aligned}
A &= \text{the largest possible area of a rectangle with perimeter 32} \\
B &= \text{the largest possible area of a rectangle with integer side lengths and perimeter 26} \\
C &= \text{the } 2015^{\text{th}} \text{ derivative of } \cos(x) \text{ with respect to } x \\
D &= \text{the } 2015^{\text{th}} \text{ derivative of } \sin(x) \text{ with respect to } x
\end{aligned}$$

Find $A + B + C + D$.